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Coupled coincidence points for two mappings in metric spaces and cone metric spaces

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Abstract

This article is concerned with coupled coincidence points and common fixed points for two mappings in metric spaces and cone metric spaces. We first establish a coupled coincidence point theorem for two mappings and a common fixed point theorem for two *w*-compatible mappings in metric spaces. Then, by using a scalarization method, we extend our main theorems to cone metric spaces. Our results generalize and complement several earlier results in the literature. Especially, our main results complement a very recent result due to Abbas et al.

1 Introduction

Throughout this article, unless otherwise specified, we always suppose that $\mathbb{N}$ is the set of positive integers and X is a nonempty set. In addition, for convenience, we denote $gx = g(x)$ for each $x \in X$ and each mapping $g : X \to X$.

Recently, Abbas et al. [1] introduced the following concept of *w*-compatible mappings:

Definition 1.1. *The mappings $g : X \to X$ and $F : X \times X \to X$ are called w-compatible if $g(F(x, y)) = F(gx, gy)$ whenever $gx = F(x, y)$ and $gy = F(y, x)$.*

Moreover, they established several coupled coincidence point theorems and common fixed point theorems for such mappings. The problem investigated in [1] is interesting. In fact, recently, the existence of coupled fixed points, coupled coincidence points, coupled common fixed points, and common fixed points for nonlinear mappings with two variables has attracted more and more attention. For example, Bhashkar and Lakshmikantham [2] investigated some coupled fixed point theorems in partially ordered sets, and they also discussed an application of their result by investigating the existence and uniqueness of the solution for a periodic boundary value problem; Sabetghadam et al. [3] extended some results in [2] to cone metric spaces; Lakshmikantham and Ćirić [4] proved several coupled coincidence and coupled common fixed point theorems for nonlinear contractive mappings in partially ordered complete metric spaces; Karapinar [5] extended some results of [4] to cone metric spaces; Zoran and Mitrović [6] considered this topic in normed spaces and established a coupled best approximation theorem; Ding et al. [7] established some coupled coincidence and coupled common fixed point theorems in partially ordered metric spaces under some generalized contraction conditions; etc.